The Millikan Oil Drop Experiment

7/10

looks good
selection of data –
too many charges.

By
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Purpose

The purpose of this experiment is to discover the quantization of electric charge, and then to measure one of these unit charges: the charge on an electron.

Introduction

This experiment is named after Robert Andrews Millikan (1868 – 1953) who devised it in the early 1900s. His goal was to "determine whether electrical charges occurred in discrete units and, if it did, whether there was such a thing as an elementary charge."[1] During a period from 1909 to 1919, Millikan performed many experiments at the University of Chicago in which he and his colleagues measured the elementary charge on the electron. After measurements on thousands of oil droplets, his data showed with great precision that electrical charge was quantized by electrons, and that charge for an electron was $e = 1.60 \cdot 10^{-19}$ Coulombs.

If a sphere falls through the air, there will be two forces acting upon it. The first, of course, is gravity: $F_{weight} = -mg$, where m is the mass of the sphere, and g is the acceleration due to gravity. Since the sphere is falling through the air, there will be some air resistance force acting upon it: $F_{air} = -Cv$, where C is some constant, and v is the velocity of the sphere. The sphere will reach some terminal velocity when $F_{weight} = -F_{air}$ and we have the equation $-Cv - mg = 0$.

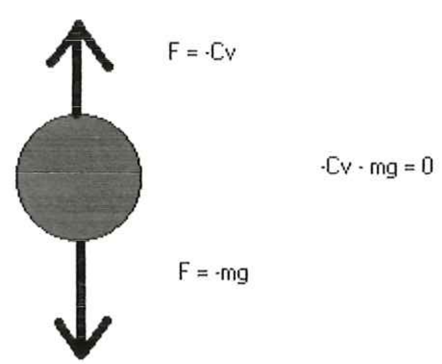


Figure 1: The Falling Sphere

The constant C for the air resistance force is not a general constant. It was first derived by Sir George Stokes to be dependent on the radius of the sphere, r, and the viscosity of the air, η, that the sphere falls through, so that $F_{air} = -Cv = -6\pi\eta rv$.

Now if an electric field is applied in the vertical direction, charged spheres will have an additional force applied to them. If the sphere has a charge q, and the electric field is of magnitude E, then the force applied to the sphere will be $F_{field} = Eq$. The sum of

[1] "Millikan Oil-Drop Apparatus", Cenco Operating Instructions: Central Scientific Company, Illinois, 1987. p. 1.

forces on the sphere now becomes $-Cv - mg + Eq = 0$. The right side of the equation remains 0 because a new terminal velocity is reached, and there is no acceleration. However, if the polarity of the field is reversed, the sphere will be accelerated in the opposite position until the forces again balance, with a new terminal velocity. One should then make a distinction between the different terminal velocities, so in summary we have:

1. $-Cv - mg = 0$
2. $-Cv_{pos} - mg + E_1q = 0$
3. $-Cv_{neg} - mg + E_2q = 0$

If we subtract equation 3 from equation 2 we get $-C(v_{pos} + v_{neg}) + (E_1-E_2)q = 0$. (Actually you get $-C(v_{pos} - v_{neg}) + (E_1-E_2)q = 0$, but since v_{neg} will be a negative value and the absolute value will be used, the sum may be used if the distinction is noted.) Since in our case $E_1 = -E_2$, we get the new equation

4. $2Eq = C(v_{pos} + v_{neg})$

Likewise, we could also subtract equation 1 from equation 2 or 3 to get

5. $Eq = C(v_{pos} + v)$
6. $-Eq = C(-v_{neg} + v)$

Note: Since absolute values are being used for E and v_{neg}, minus signs must be added for equation 6.

An experiment that measures different velocities of charged spheres in the presence and absence of an electric field should reveal the charge on those spheres. Once q is calculated for many different spheres, one may discover that the charge on those spheres is quantized by a single electron charge: $1.602177*10^{-19}$C.

Experiment

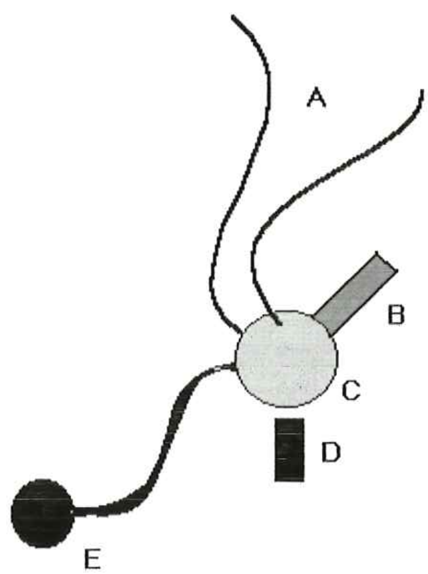


Figure 2: Overhead View of the Apparatus

A basic diagram of the apparatus is shown in figure 2. The main components are:

A. Voltage Wires
B. Light Source
C. Enclosed Cylinder
D. Microscope
E. Sphere Source

Distance Across top and bottom of cylinder: $d = 5*10^{-3}$m
Constant for spheres in air: $\eta = 1.8*10^{-5}$
Latex sphere radius: $r = 5.5*10^{-7}$m

Procedures

Best results are obtained when the spheres move slowly in an electric field. To achieve this, the applied voltage must be varied until the observed spheres move at an easily traceable rate of speed. It should not be necessary to alter this voltage throughout the rest of the experiment once a satisfying field has been achieved.

Since different spheres may hold different charges, it is desirable to obtain many measurements for each sphere. Although this is easier said than done, one should attempt to obtain several measurements without an applied field, with an applied field, and with a reversed applied field for each sphere.

Spheres should be seen moving across divisions through the microscope. Since the distance between each division is known, the spheres may be timed as they pass these divisions to determine their velocities.

Results

TABLE 1: RAW DATA
Complete Data List

uncertainties:	**0.1**	**0.1**	**0.1**	**0.1**				
			TIME (s)					
BALL	**VOLTAGE (V)**	**FALL**	**DOWN**	**UP**	**E**	**vf**	**vd**	**vu**
1	110.5	28.53	4.65	6.65	22100	3.505E-05	0.000215	0.0001504
2	108.8	30.53		5.93	21760	3.275E-05		0.0001686
3	108.8	27.5		4.06	21760	3.636E-05		0.0002463
4	-108.8	27.34	2.53	2.97	-21760	3.658E-05	0.000395	0.0003367
5	110.5	25.41		2	22100	3.935E-05		0.0005
6	110.5	31.28	1.19	2	22100	3.197E-05	0.00084	0.0005
7	110.5	27.75	2.68	3.71	22100	3.604E-05	0.000373	0.0002695
	110.5	25.93		3.41	22100	3.857E-05		0.0002933
	110.5	28.34		3.47	22100	3.529E-05		0.0002882
8	85	16.38	4.47	8.78	17000	6.105E-05	0.000224	0.0001139
9	85	26.44		6.06	17000	3.782E-05		0.000165
	85	27.81		6.41	17000	3.596E-05		0.000156

10	85	23.25	3.21	4.78	17000	4.301E-05	0.000312	0.0002092
	85	23.87		4.28	17000	4.189E-05		0.0002336
	85			4.47	17000			0.0002237
11	85	19.21		2.03	17000	5.206E-05		0.0004926
12	85	20.78		14.34	17000	4.812E-05		6.974E-05
13	77	20.44		8.47	15400	4.892E-05		0.0001181
14	77	30.72	5.13	7.06	15400	3.255E-05	0.000195	0.0001416
	77			6.25	15400			0.00016
15	77	37.03		3.22	15400	2.701E-05		0.0003106
16	77	14.88		7.16	15400	6.72E-05		0.0001397
17	77	17.69	4.19	9.53	15400	5.653E-05	0.000239	0.0001049
	77	18.47	4.97	9.47	15400	5.414E-05	0.000201	0.0001056
	77			9.06	15400			0.0001104
	77			8.59	15400			0.0001164
18	77	42.84	1.94	2.53	15400	2.334E-05	0.000515	0.0003953
19	76.7	27.19	2.05	2.66	15340	3.678E-05	0.000488	0.0003759
	76.7	24.66	1.75	2.19	15340	4.055E-05	0.000571	0.0004566
	76.7	23.84	1.87	2.56	15340	4.195E-05	0.000535	0.0003906
	76.7	27.47	1.94	2.22	15340	3.64E-05	0.000515	0.0004505
	76.7	25		2.5	15340	0.00004		0.0004
	76.7	29.69		2.53	15340	3.368E-05		0.0003953
20	76.4	19.66	4.46	8.31	15280	5.086E-05	0.000224	0.0001203
	76.4	24.53	4.82	8.16	15280	4.077E-05	0.000207	0.0001225
	76.4	21.53	4.96	7.66	15280	4.645E-05	0.000202	0.0001305
	76.4	21.53		7.72	15280	4.645E-05		0.0001295

Table 1 above shows the raw data collected. This data was then compressed by finding the average velocities for each ball and each category. The number of measurements that make up an averaged velocity in table 2 is indicated in the 'N' column after each velocity.

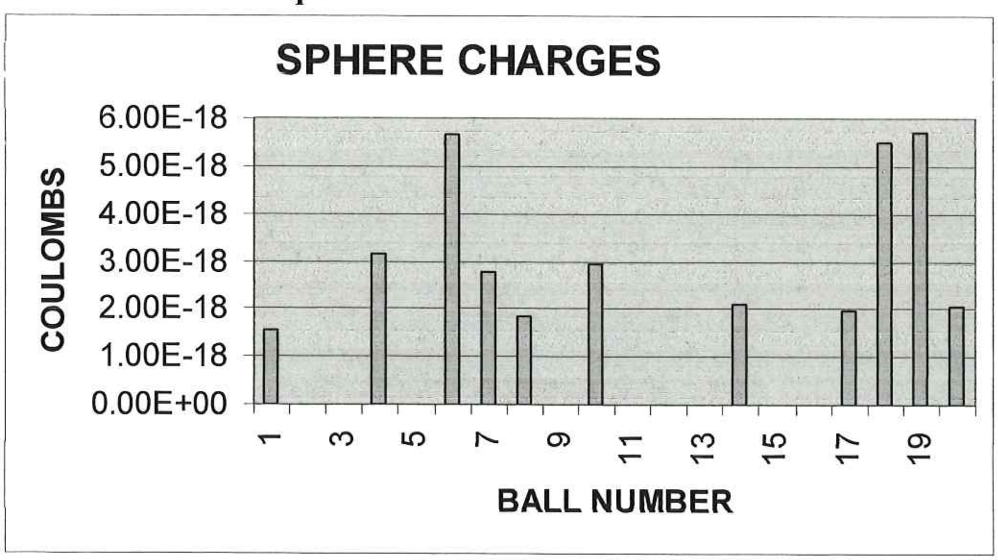
The charge on each sphere, denoted by 'q' was then determined by three different methods as suggested in table 2. These methods, 4, 5, and 6, correspond to equations 4, 5, and 6 in the introduction. This allows for an average measured charge q to be determined in the cases where the three different velocity types were managed to be measured. This should give way to slightly more accuracy.

TABLE 2: Charge Analysis

								METHOD:			
		Velocities in m/s						4	5	6	
BALL	**E (V/m)**	**Vfall**	**N**	**Vdown**	**N**	**Vup**	**N**	**q**	**q**	**q**	**avg q**
1	22100	3.51E-05	1	2.15E-04	1	1.50E-04	1	1.54E-18	1.52E-18	1.57E-18	1.5427E-18
2	21760	3.28E-05	1			1.69E-04	1			1.73E-18	
3	21760	3.64E-05	1			2.46E-04	1			2.42E-18	
4	21760	3.66E-05	1	3.95E-04	1	3.37E-04	1	3.14E-18	3.08E-18	3.20E-18	3.1383E-18
5	22100	3.94E-05	1			5.00E-04	1			4.55E-18	
6	22100	3.20E-05	1	8.40E-04	1	5.00E-04	1	5.66E-18	6.83E-18	4.49E-18	5.6583E-18
7	22100	3.66E-05	3	3.73E-04	1	2.83E-04	3	2.77E-18	2.84E-18	2.70E-18	2.7711E-18
8	17000	6.11E-05	1	2.24E-04	1	1.14E-04	1	1.85E-18	1.79E-18	1.92E-18	1.8528E-18
9	17000	3.78E-05	2			1.60E-04	2			2.18E-18	
10	17000	4.24E-05	2	3.12E-04	1	2.22E-04	3	2.93E-18	2.95E-18	2.90E-18	2.9265E-18
11	17000	5.21E-05	1			4.93E-04	1			5.98E-18	
12	17000	4.81E-05	1			6.97E-05	1			1.29E-18	
13	15400	4.89E-05	1			1.18E-04	1			2.02E-18	
14	15400	3.26E-05	1	1.95E-04	1	1.50E-04	2	2.09E-18	1.97E-18	2.22E-18	2.0913E-18
15	15400	2.70E-05	1			3.11E-04	1			4.09E-18	
16	15400	6.72E-05	1			1.40E-04	1			2.51E-18	
17	15400	5.53E-05	2	2.18E-04	2	1.09E-04	4	1.98E-18	1.98E-18	1.99E-18	1.984E-18
18	15400	2.33E-05	1	5.15E-04	1	3.95E-04	1	5.52E-18	5.96E-18	5.07E-18	5.5173E-18
19	15340	3.80E-05	6	5.26E-04	4	4.09E-04	6	5.69E-18	5.93E-18	5.44E-18	5.686E-18
20	15280	4.58E-05	4	2.11E-04	3	1.26E-04	4	2.05E-18	2.01E-18	2.09E-18	2.0532E-18

The question now is whether the calculated charges are quantized, and if so, what is the basic unit of charge. One might expect the averaged q values in the last column of table 2 to be most accurate, and attempt to examine these first:

GRAPH 1: The best 'q's?



A quick look at graph 1 gives the impression that the charges on the little spheres were quantized. If we called the basic charge e, it would appear that three different groups of charges were observed: The first group with total charge found by xe (balls 1, 8, 14, 17, and 20), where x is some integer and e is the basic charge. The second may have total charge xe + e (balls 4, 7, and 10), and the third group would appear to have total charge found by xe + 3e (balls 6, 18, and 19). The following table gives the results when solving for e:

TABLE 3:Can it be that this is e?

Looks like 3 categories:

	x	x+e	x+3e	
	1.54E-18	3.13E-18	5.66E-18	
	1.85E-18	2.77E-18	5.52E-18	
	2.09E-18	2.93E-18	5.69E-18	
	1.98E-18			
	2.05E-18			
avg:	**1.90E-18**	**2.94E-18**	**5.62E-18**	
				% discrepency:
	e:	1.04E-18	1.24E-18	16.1290323

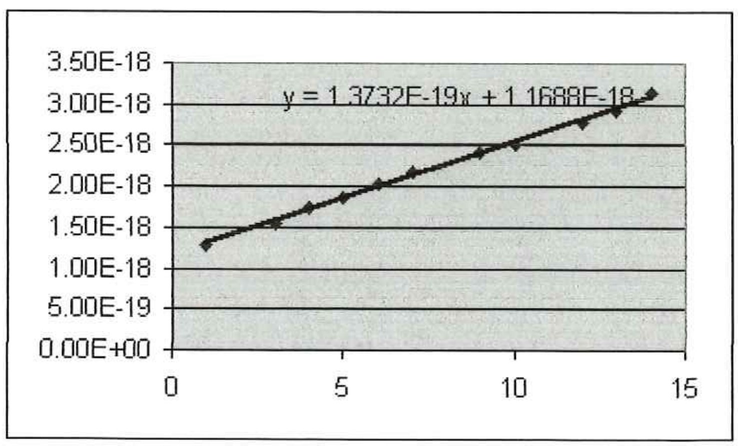
The average value for the basic charge unit was found here to be $1.14*10^{-18}$C. This first result for e should be disregarded for two reasons. The first is that upon closer inspection of the data, there appears to be even smaller charge steps amongst the spheres. Since multiple methods were used to obtain accurate data (many measurements attempted on each sphere, and three different methods of calculating q for each sphere), there should not be such a broad range of charges in each of the above three groups. (There is a 30% discrepancy between the highest and lowest charges in the first group!) The second reason for discarding this value of e, is of course, insight into the accepted value of e, which is almost an entire order of magnitude smaller than current calculated value here.

TABLE 4: A Better e

q	spaced q
1.29E-18	1.29E-18
1.54E-18	
1.73E-18	**1.54E-18**
1.85E-18	1.73E-18
1.98E-18	**1.85E-18**
2.02E-18	2.04E-18*
2.05E-18	2.18E-18
2.09E-18	
2.18E-18	2.42E-18
2.42E-18	2.51E-18
2.51E-18	
2.77E-18	**2.77E-18**
2.93E-18	**2.93E-18**
3.13E-18	**3.13E-18**

* --> averaged value

GRAPH 2: Finding a Trend line



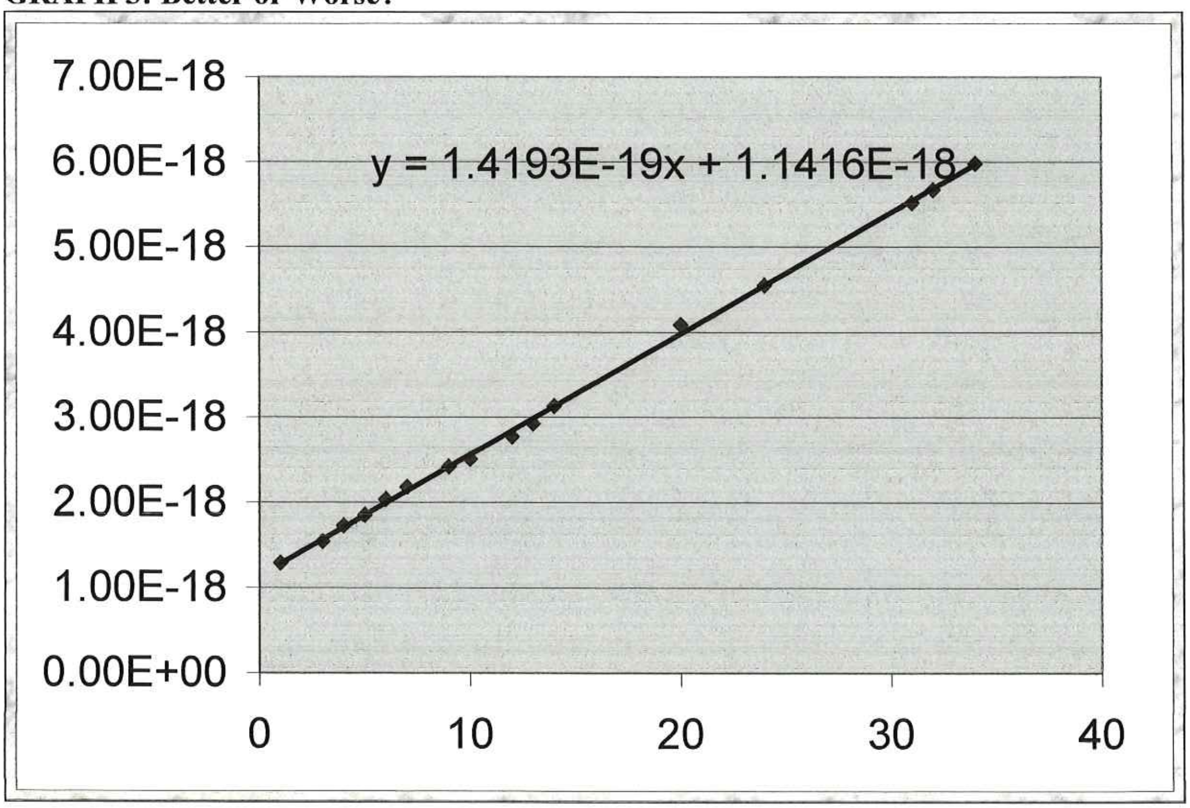
14.29162"--> % discrepancy with accepted value.

Table 4 lists the sorted charges, leaving out the six highest values. The second column is entitled "spaced q" because as a pattern of quantization emerges, certain values appear to have been missed. For this reason, the six highest charge values have been left out. The spaces are in a sense guesses, and there would have been too much guesswork with the entire list. How much spacing should be inserted amidst the higher values when trying to remain somewhat objective and non-manipulating?

Graph 2 results from the spaced charges. If $Q = xe$, then the slope of the trend line should deliver our sought after e. Indeed, the result $e = 1.3732*10^{-19}C$ is in great agreement with the accepted value of $e = 1.602177*10^{-19}C$. The 14% discrepancy between the experimental and accepted value may seem gross, however in consideration of the tediousness and "roughness" of the experiment, the agreement should in fact be seen as tremendous! Sources of error will be included in the discussion.

Out of curiosity, one might want to find a trend line on a plot of the complete charge list. Graph 3, below, was constructed with the rest of the six charges included, with guessed spacing.

GRAPH 3: Better or Worse?



Including all the charges gives $e = 1.4193*10^{-19}C$. This value has a discrepancy of 11% with the accepted value. The improved result is debatable, as the large inserted spaces may be viewed as "forcing the right answer".

TABLE 5: One More Inspection!

spaced q	n
1.29E-18	9.39
1.54E-18	11.23
1.73E-18	12.60
1.85E-18	13.49
2.04E-18	14.83
2.18E-18	15.88
2.42E-18	17.62
2.51E-18	18.28
2.77E-18	20.18
2.93E-18	21.31
3.13E-18	22.81

Table 5 is a reminder of the theory of the experiment, and an indication of its shortcomings. Remember that the basis of this lab is charge quantization. The charges on the spheres were expected to be integer multiples of a basic unit charge e. Column one in table 5 lists several charges found on several spheres. The second column is the sphere charge divided by the experimental basic charge e. Column 2 should then be expected to be a list of integers. Use of the accepted value of e does not improve this. This should be an indication also of the uncertainty of the experiment.

Discussion

While the theory behind this lab seems sound, it must be mentioned that performing the lab was aggravating and unsatisfying. Great patience is required to obtain results, as the spheres are difficult to track for any length of time. This is where the weary eyes and headaches begin. Despite efforts to make precise measurements and multiple precise measurements, the latex spheres seemed to be purposely uncooperative. ?

Two persons are required to perform the experiment. One must watch the movement of the spheres, signalling the other when a sphere passes the desired gridlines. The second person uses these signals to time the motion of the spheres so that their velocities may be determined. The first person then introduces measurement error in judgement of when a sphere passes a gridline. The sphere introduces error by not revealing whether it is travelling the assumed vertical direction only, or whether there is any "y-component" or depth to its motion. The second person introduces "signal reaction and thumb control" error in the use of the stopwatch.

1st person should time also to elliminat

There is also yet another source of error. The constant C was based on the notion that the measured charges were on single spheres of radius $r = 5.5*10^{-7}$m. However table 5 indicates that the number of electrons on these spheres ranged from 9 to 22. It is certainly conceivable that measurements were made unknowingly on 'clumped spheres'. If this were the case throughout the lab, the value of r used must have in reality varied, and the calculated constant C used for all charge calculations was not at all accurate. It would have been the minimum C used when a larger value was required.

Free fall data should reveal charge.

With so many sources of gross error, it would seem that the more measurements that would be made would be a greater indication of continuous charge, rather than quantization.

Upon analysing the data, the experiment does *somewhat* support the quantization theory of charge, but one must agree that this lab is certainly not a practical method for determining e.

Conclusions

The purpose of this lab was to verify quantization of charge, and to determine the charge on an electron. Despite tedious efforts for plentiful data and accurate results, quantization was only suggested, and not overly obvious. Nonetheless, the charge on an electron was determined to be $e = 1.3732*10^{-19}$C, 14% discrepancy with the accepted value.

References

"Millikan Oil-Drop Apparatus", Cenco Operating Instructions: Central Scientific Company, Illinois, 1987.

Serway, Raymond A., PHYSICS for Scientists and Engineers, Saunders College Publishing, Montreal, 1992.